2007

Green River

Regional Educational Cooperative

Grade 8

Mathematics

Constructed Response

Measured Progress

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Standard: MA-08-5.1.02: Patterns, Relations and Functions

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Note: Each item is aligned to a standard, but does not necessarily measure the entire standard.

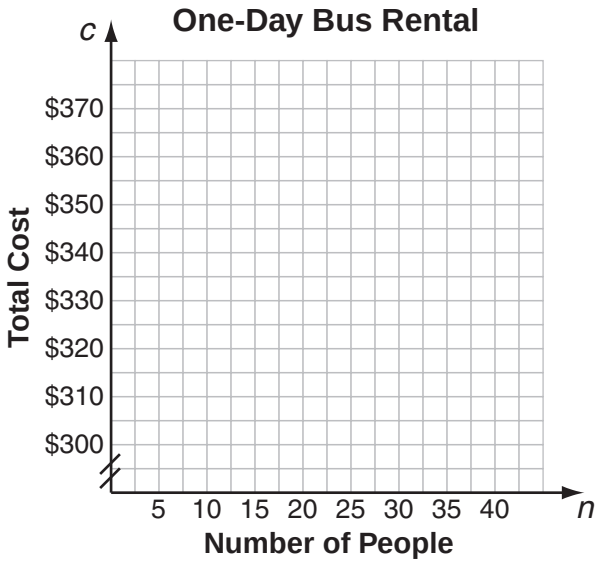
Item 1 One-Day Bus Rental

Standard: MA-08-5.1.02: Patterns, Relations, and Functions — Students will represent, analyze, and generalize simple first and second degree relationships using tables, graphs, words, and algebraic notations, and will apply the relationships to solve real-world and mathematical problems.

Bloom's Taxonomy: a. Knowledge
b. Comprehension
c. Knowledge

Depth of Knowledge: Level 2

1. Mr. Bennett's class is renting a bus for a field trip. The bus company charges $300 for a one-day rental plus $2 for each person who rides the bus.
   a. Create a table that shows the cost for a one-day rental when transporting the following groups of people:
      - a group of 10 people,
      - a group of 20 people, and
      - a group of 30 people.
   b. Write an equation that shows the relationship between the total cost, *c*, of a one-day bus rental and the number, *n*, of people transported.



   c. Graph the equation you wrote in part b.

One-Day Bus Rental

Scoring Guide

Score	Description
4	The student response demonstrates an exemplary understanding of the concepts involved in representing a simple first-degree relationship using tables, graphs, and algebraic notations.
3	The student response demonstrates a good understanding of the concepts involved in representing a simple first-degree relationship using tables, graphs, and algebraic notations. Although there is significant evidence that the student was able to recognize and apply the concepts involved, some aspect of the response is missing or flawed. As a result the response merits 3 points.
2	The student response demonstrates a fair understanding of the concepts involved in representing a simple first-degree relationship using tables, graphs, and algebraic notations. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.
1	The student response demonstrates a minimal understanding of the concepts involved in representing a simple first-degree relationship using tables, graphs, and algebraic notations.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

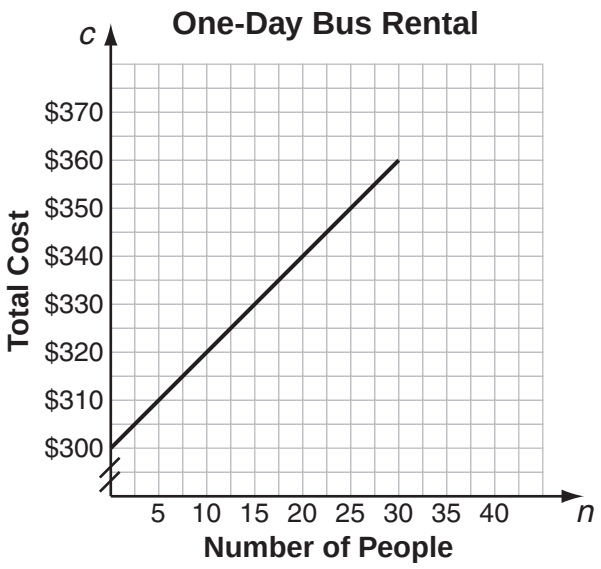
Sample Response:

Part a:

Group size	Cost
10	**$320**
20	**$340**
30	**$360**

Part b: $c = 2n + 300$

Part c:



Sample Student Responses

One-Day Bus Rental

EQUATION: $C = 300 + 2n$

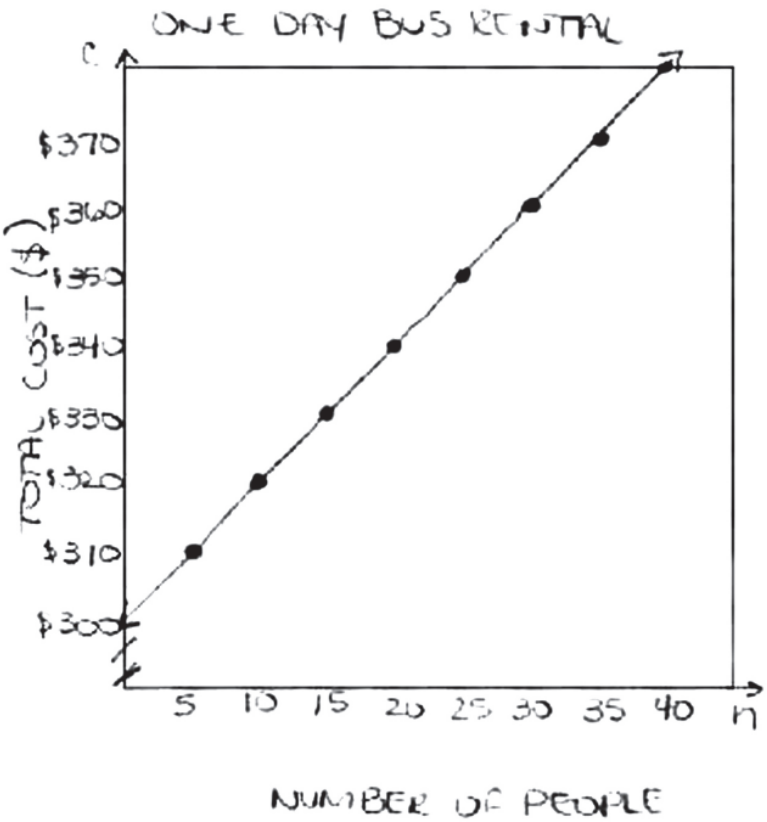
$C = 300 + 2(10) = 320$

$C = 300 + 2(20) = 340$

$C = 300 + 2(30) = 360$

TABLE

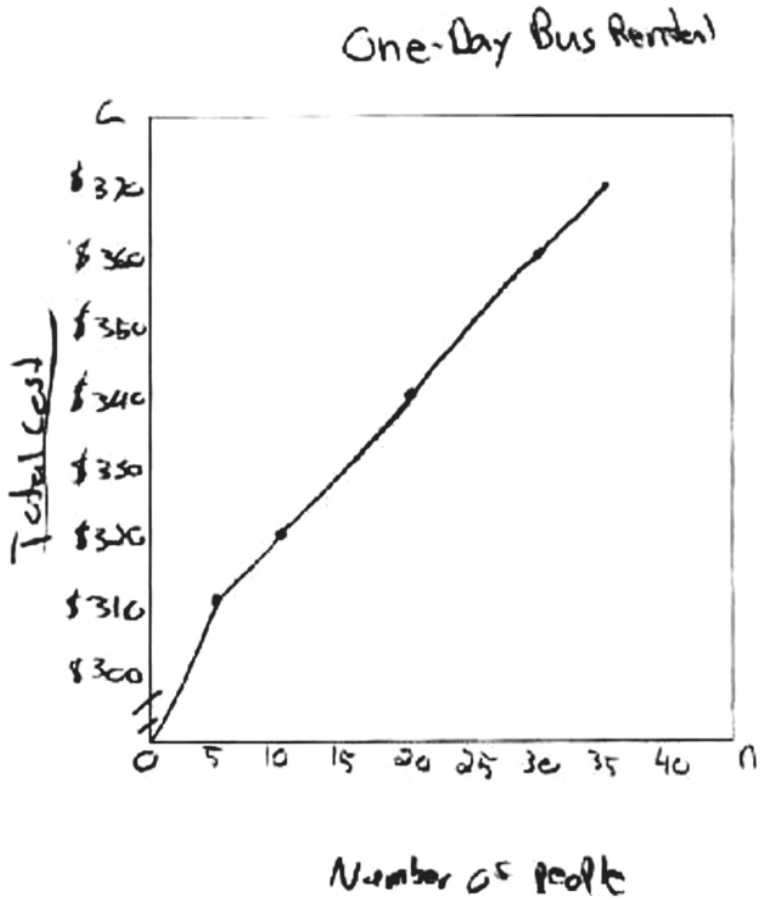
# OF PEOPLE	COST (1 DAY)
10	320
20	340
30	360



Score Point: 4

	# of people (n)	Bus rental (B)	Cost of people	cost per day (C=2n+300)
group1	10	300	2n	320
group2	20	300	2n	340
group3	30	300	2n	360

$C = 2n + 300$



Score Point: 3

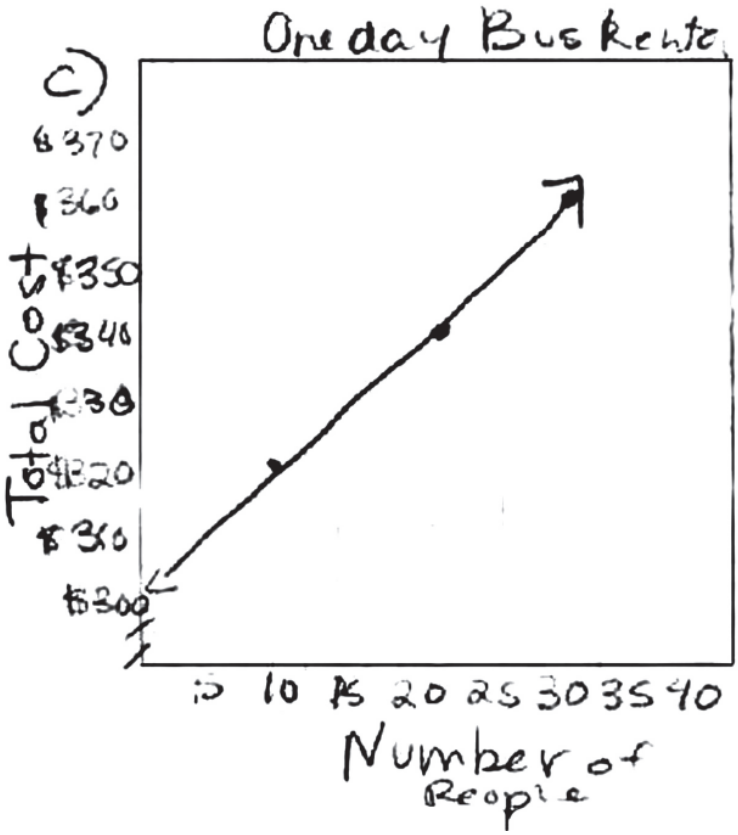
a) 10 people
$320

20 people
$340

30 people
$360

B) C = 300 + 2 × 10
C = 300 + 2 × 20
C = 300 + 2 × 30

c)

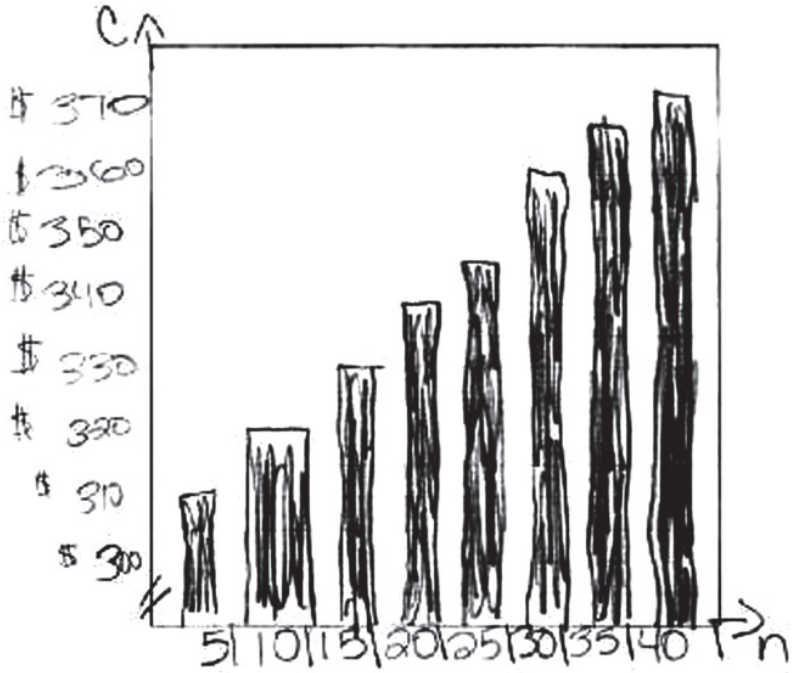


Score Point: 2

a.

People going	10	20	30
Per person	$20	$40	$60

b. $300c \div n + 2$



Score Point: 1

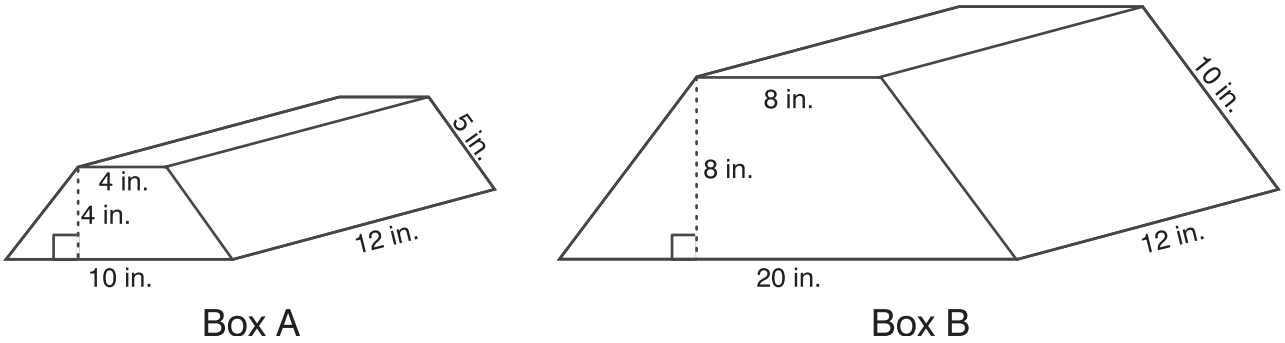
Item 2 Boxworks Company

Standard: MA-08-2.1.04: Measuring Physical Attributes — Students will apply formulas to determine the volume of right rectangular prisms in real-world problems.

Bloom's Taxonomy: Application

Depth of Knowledge: Level 3

2. Use the diagrams below to answer the question.



The Boxworks Company has been commissioned to produce the two boxes shown above.

Each box is a right prism. The ends of both boxes are isosceles trapezoids; all of the other faces are rectangles. The dimensions of the boxes are given in the diagrams above.

a. If there are no overlapping seams, how many square inches of material are needed to make Box A (what is the total surface area)? Show or explain how you found your answer.

b. As indicated in the diagrams, the lengths of the two boxes are the same but all other dimensions of Box B are two times greater than those of Box A. What is the ratio of the volumes of the two boxes? Justify your answer mathematically.

Boxworks Company

Scoring Guide

Score	Description
4	The student demonstrates a thorough understanding of the concepts of surface area and volume of right prisms by correctly finding the surface area of a right trapezoidal prism and accurately determining the ratio between the volumes of two right trapezoidal prisms.
3	The student demonstrates a general understanding of the concepts of surface area and volume of right prisms by finding the surface area of a right trapezoidal prism and determining the ratio between the volumes of two right trapezoidal prisms with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback.
2	The student demonstrates a basic understanding of the concepts of surface area and volume of right prisms by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks.
1	The student demonstrates a minimal understanding of the concepts of surface area and volume of right prisms. The response indicates that the student would require significant instruction to complete the tasks.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Sample Response:

Part a: $\frac{4+10}{2}(4)(2) + (12)(5)(2) + 4(12) + 10(12) = 344$ square inches

Part b: The height and the width of the bases were both multiplied by 2, so the area of the base increased by a factor of 4. Since the length doesn't change and volume is the area of the base times the height, the volume is increased by a factor of 4 too. 4 : 1

OR

Volume of Box A: $\frac{4+10}{2}(4)(12) = 336$ cubic inches

Volume of Box B: $\frac{8+20}{2}(8)(12) = 1344$ cubic inches 336 : 1344 OR 1 : 4

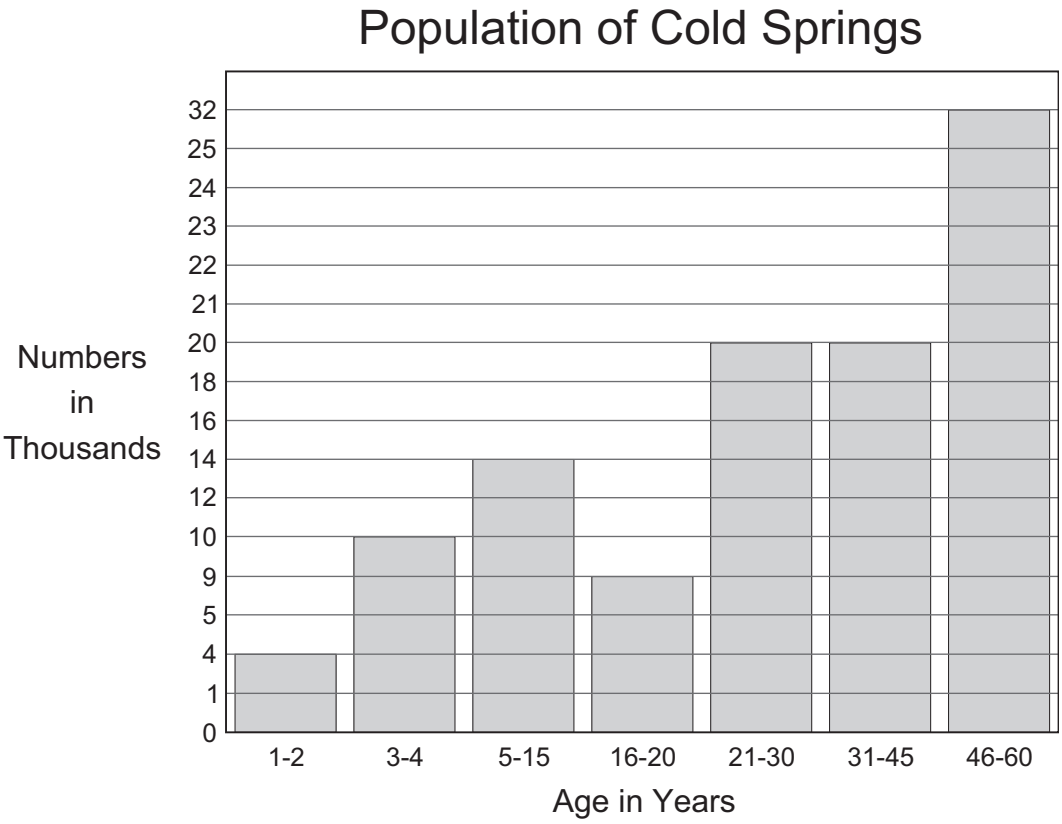
Item 3 Population Histogram

Standard: MA-08-4.1.04: Presentations of Data Sets — Students will construct data displays (Venn diagrams, tables, line graphs, stem-and-leaf plots, circle graphs, scatter plots); explain why the type of display is appropriate for the data and explain how misleading representations affect interpretations and conclusions about data (e.g., changing the scale on a graph).

Bloom's Taxonomy: Application

Depth of Knowledge: Level 3

3. Graphs are sometimes made to give misleading information. Sam made the graph below to convince the town's government that things look bad for Cold Springs. He says there are too few people in the younger age ranges.



a. Describe two things wrong with Sam's graph.

b. Make a new graph on graph paper that correctly represents the data.

c. Make a conclusion about the population of Cold Springs based on the graph you drew in part b.

Population Histogram

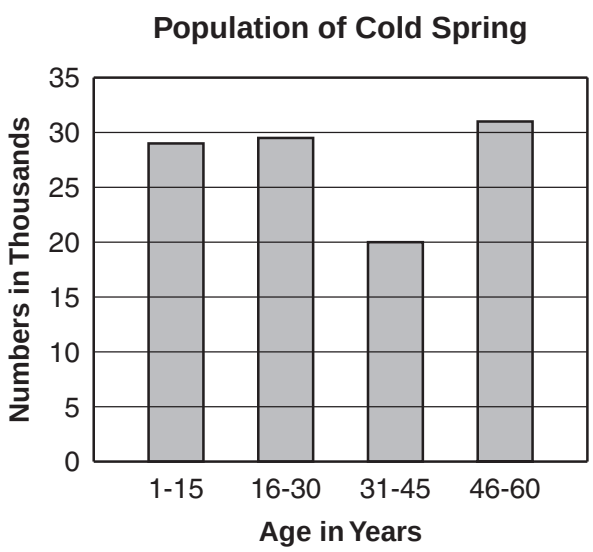
Scoring Guide

Score	Description
4	The student demonstrates a thorough understanding of histograms by appropriately critiquing an incorrectly constructed histogram, constructing a correct histogram, and using it to draw an accurate conclusion.
3	The student demonstrates a general understanding of histograms by critiquing an incorrectly constructed histogram, constructing a histogram, and using it to draw a conclusion with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback.
2	The student demonstrates a basic understanding of histograms by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks.
1	The student demonstrates a minimal understanding of histograms. The response indicates that the student would require significant instruction to complete the tasks.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Sample Response:

Part a: The horizontal scale is not even; 1–2 is only 1 year apart while 46–60 is 14 years apart.
The vertical scale is not even; 0–1 only increases by 1 but 1–4 increases by 3.

Part b:



Population Histogram

Part c: The 31–45-year-old group has the lowest population in Cold Springs.
The 46–60-year-old group has the greatest population in Cold Springs.
The 1–15-year-old age group and the 16–30-year-old age group have a difference of only 1000 people.
There are more people 30 years old and younger than those older than 30 years old in Cold Springs.

Item 4 Toothpick Pattern

Standard: MA-08-5.1.02: Patterns, Relations and Functions — Students will represent, analyze and generalize simple first and second degree relationships using tables, graphs, words and algebraic notations, and will apply the relationships to solve real-world and mathematical problems.

Bloom's Taxonomy: Application

Depth of Knowledge: Level 3

4. Use the pattern below to answer the question.

This pattern is made from toothpicks.

a. Draw the next step in the pattern.

b. How many toothpicks in all will be needed for **Step 5** of the pattern? Show or explain how you found your answer.

c. How many toothpicks in all will be needed for **Step 12** of the pattern? Show or explain how you found your answer.

d. Write a formula, using *n*, that will show how many toothpicks are needed to build **Step *n*** in the pattern.

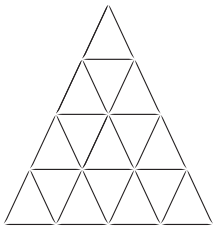
Toothpick Pattern

Scoring Guide

Score	Description
4	The student demonstrates a thorough understanding of a toothpick pattern by correctly extending the pattern, correctly determining the number of toothpicks in various steps, and accurately writing an algebraic expression for the number of toothpicks in the *n*th step.
3	The student demonstrates a general understanding of a toothpick pattern by extending the pattern, determining the number of toothpicks in various steps, and writing an algebraic expression for the number of toothpicks in the *n*th step with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback.
2	The student demonstrates a basic understanding of a toothpick pattern by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks.
1	The student demonstrates a minimal understanding of a toothpick pattern. The response indicates that the student would require significant instruction to complete the tasks.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Sample Response:

Part a:



Step 4

Part b: $3(5 + 4 + 3 + 2 + 1) = 45$

OR

$3\left(\frac{6 \times 5}{2}\right) = 3 \times 3 \times 5 = 45$

Toothpick Pattern

Part c: $3(12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) = 234$

OR

$3\,\frac{(13 \times 12)}{2} = 3 \times 13 \times 6 = 234$

Part d: $3\frac{(n+1)n}{2}$

OR

$1.5(n^2 + n)$

OR

$1.5n^2 + 1.5n$

Item 5 Canoe Rental Fees

Standard: MA-08-5.1.02: Patterns, Relations and Functions — Students will represent, analyze and generalize simple first and second degree relationships using tables, graphs, words and algebraic notations, and will apply the relationships to solve real-world and mathematical problems.

Bloom's Taxonomy: Comprehension

Depth of Knowledge: Level 2

5. The River Bend Store charges the following fees for canoe rentals.

1 hour.....$13.00

3 hours.....$23.00

4 hours.....$28.00

a. On graph paper, draw a line graph that shows the relationship between the rental time and the cost for renting a canoe. Put the time on the *x*- (horizontal) axis.

b. The relationship between hours and rental cost is linear. If you want to rent a canoe from the River Bend Store for 7 hours, how much will it cost? Explain or show how you found your answer.

c. Explain what information can be found about the store's canoe rental rates by determining (1) the slope and (2) the *y*-intercept of the graph in part a.

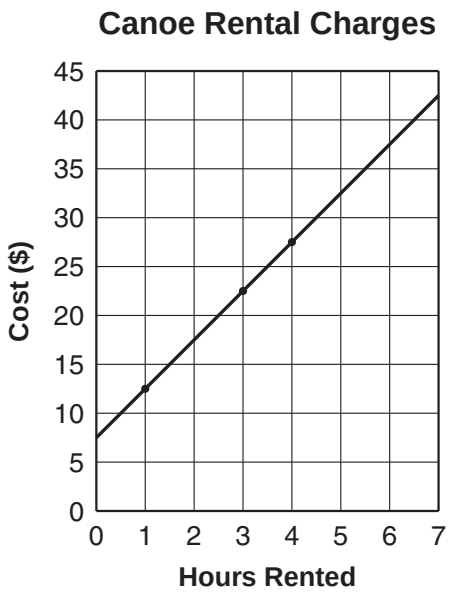
Canoe Rental Fees

Scoring Guide

Score	Description
4	The student demonstrates a thorough understanding of basic linear algebraic concepts by accurately constructing a graph, correctly making a prediction, and accurately explaining the meaning of the slope and an intercept relative to the given context.
3	The student demonstrates a general understanding of basic linear algebraic concepts by constructing a graph, making a prediction, and explaining the meaning of the slope and an intercept relative to the given context with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback.
2	The student demonstrates a basic understanding of basic linear algebraic concepts by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks.
1	The student demonstrates a minimal understanding of basic linear algebraic concepts. The response indicates that the student would require significant instruction to complete the tasks.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Sample Response:

Part a:



Canoe Rental Fees

Part b: \$43 Look at the 7 on the x-axis and go up until you hit the line. Then look left until you see the y-value.
OR
It's \$5 for every additional hour so $28 + 3(5) = 43$.
OR
$y = 5x + 8 \quad 5(7) + 8 = \$43$

Part c: slope: set fee for each hour beyond the first hour
y-intercept: base charge
OR
slope: \$5 per additional hour; y-intercept: \$8 base fee