2007

# Green River

## *Regional Educational Cooperative*

Grade 9

Mathematics

Constructed Response

Measured Progress

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**Note:** Each item is aligned to a standard, but does not necessarily measure the entire standard.

## Item 1 Tickets

Standard: MA-HS-5.3.03: Equations and Inequalities — Students will model, solve, and graph first-degree, two-variable equations and inequalities in real-world and mathematical problems.

Bloom's Taxonomy: Application

Depth of Knowledge: Level 2

1. The drama club put on a play on Friday night. Some tickets were sold in advance and some tickets were sold at the door. A total of 300 tickets were sold.

   a. Using $x$ to represent the number of tickets sold in advance, write an expression that represents the number of tickets sold at the door.

   The club charged \$3 for tickets sold in advance and \$4 for tickets sold at the door. The total amount of money collected from tickets was \$1072.

   b. Again using $x$ to represent the number of tickets sold in advance, write **one** equation that can be used to find the number of \$3 tickets and the number of \$4 tickets sold. Your equation should contain no variables except $x$.

   c. How many \$3 tickets and how many \$4 tickets were sold? Show your work or explain how you found your answer.

# Tickets

## Scoring Guide

| Score | Description |
|---|---|
| **4** | The student response demonstrates an exemplary understanding of the concepts involved in modeling and solving first-degree, two-variable equations in a real-world problem. |
| **3** | The student response demonstrates a good understanding of the concepts involved in modeling and solving first-degree, two-variable equations in a real-world problem. Although there is significant evidence that the student was able to recognize and apply the concepts involved, some aspect of the response is missing or flawed. As a result the response merits 3 points. |
| **2** | The student response demonstrates a fair understanding of the concepts involved in modeling and solving first-degree, two-variable equations in a real-world problem. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points. |
| **1** | The student response demonstrates a minimal understanding of the concepts involved in modeling and solving first-degree, two-variable equations in a real-world problem. |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

## Additional Notes

In part c, response does not have to use an algebraic solution in order to receive full credit.

## Sample Response:

Part a: $300 - x$

Part b: $1072 = 3x + 1200 - 4x$

Part c: $-128 = -x$

$x = 128$

So 128 tickets were sold in advance and $300 - 128 = 172$ were sold at the door.

# Sample Student Responses

## Tickets

(a) # sold at door = 300 − x

(b) 1072 = 3x + 4(300 − x)

(c) 1072 = 3x + 4(300 − x)

1072 = 3x + 1200 − 4x

1072 = −x + 1200

−1200 −1200

(−128 = −x)−1

128 = x

300 − 128 = 172

128 - $3 tickets

172 - $4 tickets

Score Point: 4

a. $300 - x$ = tickets sold at the door

b. $1072 - 3x$ ← tickets in advance once found will be able to find # of tickets sold at the door

c. $x + y = 300$

$3x + 4y = 1072$

$x = 300 - y$

$3(300 - y) + 4y = 1072$

$900 - 3y + 4y = 1072$

$-900 \qquad\qquad -900$

$y = 172$

172 tickets @ door were sold

$300 - 172 = 128$

128 tickets were sold in advance.

Score Point: 3

a. $300 - x = t$

b. $1072 - 3x = x - 4(1072 - 3x)$

c. Use the above equation

$1072 - 3x = x - 4(1072 - 3x)$

$1072 - 3x = x - 4288 - 12x$

$+12x$

$1072 - 3x = 13x - 4288$

$+4288 \qquad +4288$

$5360 - 3x = 13x$

$+3x \quad +3x$

$\frac{5360}{16} = \frac{16x}{16}$

$335 = x$

335 were \$3 tickets

737 were \$4 tickets

Score Point: 2

a.) $300 - x =$ # of tickets sold at door

b.) $300 - 3x = 4x$

c.) 60 tickets were sold for each

$$300 - 3x = 4x$$
$$+3x \quad +3x$$
$$300 = 5x$$
$$60 = x$$

Score Point: 1

## Item 2 Hot Dog and Hamburger Sales

**Standard:** MA-HS-4.2.03: Characteristics of Data Sets — Students will: identify an appropriate curve of best fit (linear, quadratic, exponential) for a set of two-variable data; determine a line of best fit equation for a set of linear two-variable data and apply a line of best fit to make predictions within and beyond a given set of two-variable data.

**Bloom's Taxonomy:** Application

**Depth of Knowledge:** Level 2

2. The Varsity Club sells hot dogs and hamburgers at school home games. Club members are trying to decide how many hamburgers and hot dogs to order for the homecoming game. They collected the following data on sales at the previous ten games.

| GAME | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of hot dogs sold | 30 | 41 | 49 | 60 | 71 | 39 | 20 | 46 | 61 | 64 |
| Number of hamburgers sold | 60 | 63 | 70 | 80 | 76 | 73 | 65 | 77 | 81 | 73 |

a. On your graph paper, construct a scatter plot by graphing these data. Put the number of hot dogs sold along the horizontal axis and the number of hamburgers sold along the vertical axis. Draw a line of best fit on your graph.

b. For the homecoming game, the Varsity Club traditionally has 200 sales, some of which are hamburgers and some of which are hot dogs. Based on this information and the data from the previous ten games, predict how many hamburgers the club can plan to sell. Explain how you made your prediction.

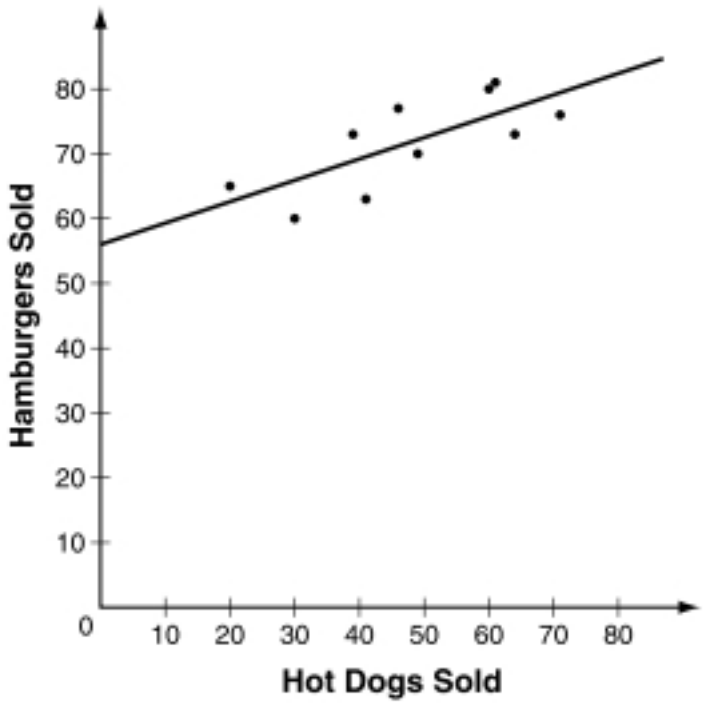
## Hot Dog and Hamburger Sales

**Scoring Guide**

| Score | Description |
|---|---|
| **4** | The student demonstrates a thorough understanding of scatter plots by correctly constructing a scatter plot, sketching an approximate line of best fit, and making a reasonable prediction. |
| **3** | The student demonstrates a general understanding of constructing a scatter plot, sketching an approximate line of best fit, and making a reasonable prediction with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback. |
| **2** | The student demonstrates a basic understanding of scatter plots by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks. |
| **1** | The student demonstrates a minimal understanding of scatter plots. The response indicates that the student would require significant instruction to complete the tasks. |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

**Sample Response:**

Part a:



## Hot Dog and Hamburger Sales

Part b: They should plan to sell about 133 hamburgers and 67 hot dogs. I based my prediction on the equation of my line of best fit. The slope is about $\frac{1}{3}$ and the $y$-intercept is about 55, so the equation of the line is $y = \frac{1}{3}x + 55$. So if $x = 100$, $y$ is about 88. If $x = 109$, $y = 91$. $109 + 91 = 200$. So I predict they would sell about 91 hamburgers.
OR
Student may use the line of best fit to geometrically predict the number of hamburgers sold.

Note: Students are only sketching the line of best fit, so the lines will vary. The prediction in part b should be based on the line of best fit the student drew.

## Item 3 Defective Parts

Standard: MA-HS-4.4.01: Probability — Students will: determine theoretical and experimental (from given data) probabilities; make predictions and draw inferences from probabilities; compare theoretical and experimental probabilities and determine probabilities involving replacement and non-replacement.

Bloom's Taxonomy: Application

Depth of Knowledge: Level 3

3. A manufacturing company orders parts from two different suppliers. The rate of defective parts from Supplier P is 1 in 250, and the rate of defective parts from Supplier Q is 1 in 450.
   a. The company orders some parts from each supplier. Can you tell if more defective parts come from Supplier P or from Supplier Q? If so, state which one and explain how you can tell. If not, explain what additional information would be needed.

   b. If the company orders 10,000 parts from Supplier P, how many parts would have to be ordered from Supplier Q to have the same expected number of defective parts? Show or explain your answer.

   c. If the company uses one part from each supplier in a machine, what is the probability that both parts in a single machine will be defective? Show or explain how you found your answer.

## Defective Parts

### Scoring Guide

| Score | Description |
|---|---|
| **4** | The student demonstrates a thorough understanding of a real-life problem involving probabilities by correctly applying given probabilities to a situation and accurately calculating a compound probability. |
| **3** | The student demonstrates a general understanding of a real-life problem involving probabilities by applying given probabilities to a situation and calculating a compound probability with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback. |
| **2** | The student demonstrates a basic understanding of a real-life problem involving probabilities by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks. |
| **1** | The student demonstrates a minimal understanding of the problem. The response indicates that the student would require significant instruction to complete the tasks. |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

### Sample Response:

Part a: More defective parts are likely to come from P because you would only need to order about 250 to get 1 defective, but with Q you would only find 1 defective part in 450.

OR

Probably more from P because $\frac{1}{250} > \frac{1}{450}$

OR

It's tough to know because we don't know how many are being ordered. If they only order a few from each supplier, anything could happen. If they order a lot, the probability would help you figure out the number of defective parts.

Part b: $\frac{1}{250}(10,000) = 400$

$400 = \frac{1}{450}x$ $\qquad x = 18,000$

Part c: $\frac{1}{250} \times \frac{1}{450} = \frac{1}{112,500}$

## Item 4 Speeding Fines

Standard: MA-HS-5.3.01: Equations and Inequalities — Students will model, solve and graph first degree, single variable equations and inequalities, including absolute value, based in real-world and mathematical problems and graph the solutions on a number line.

Bloom's Taxonomy: Application

Depth of Knowledge: Level 3

4. In one state, the fine for speeding on highways where the speed limit is 70 miles per hour (mph) is calculated using the formula $\boldsymbol{F = 10(S - 70) + 35}$, where $\boldsymbol{S}$ represents the car's speed in mph and $\boldsymbol{F}$ is the dollar value of the fine.
   a. Ted was stopped for traveling 80 miles per hour on the highway. What should his fine be? Show your work or explain your answer.
   b. Janice received a $215 speeding fine. How fast was she going? Show your work or explain how you arrived at your answer.
   c. Would it be possible to receive a fine for $35 for speeding on an interstate highway with a speed limit of 70 mph? Show your work or explain your reasoning.

## Speeding Fines

**Scoring Guide**

| Score | Description |
|---|---|
| **4** | The student demonstrates excellent problem solving skills by correctly analyzing and solving real-world problems by evaluating linear expressions, solving linear equations, and interpreting a linear relationship in a real-life situation. |
| **3** | The student demonstrates good problem solving skills by analyzing and solving real-world problems by evaluating linear expressions, solving linear equations, and interpreting a linear relationship in a real-life situation with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback. |
| **2** | The student demonstrates basic problem solving skills by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the task. |
| **1** | The student demonstrates minimal problem solving skills. The response indicates that the student would require significant instruction to complete the task. |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

**Sample Response:**

Part a: The fine should be $10(80 - 70) + 35 = 100 + 35 = \$135$

Part b: $215 = 10(S - 70) + 35$

$215 = 10S - 700 + 35$

$215 = 10S - 665$

$880 = 10S$

$88 = S$

She was going 88 mph.

Part c: It would not be possible because if the fine was \$35, the person would have to be going 70 mph, which is the speed limit and there would not be a fine.

## Item 5 Hiring a Popular Band

**Standard:** MA-HS-5.3.04: Equations and Inequalities — Students will model, solve and graph systems of two linear equations in real-world and mathematical problems.

**Bloom's Taxonomy:** Application

**Depth of Knowledge:** Level 3

5. The junior class is planning a concert as a fund-raiser. The class plans to hire one of two popular bands: the Cautious Crew, which charges $250 plus 12% of total ticket sales, or the Muck Rockets, which charges a flat rate of $500. Tickets will cost $5.00 each. The class has no expenses to pay beyond the cost of the band.

   a. Calculate the junior class's profit with each of the two bands, assuming that the class sells 300 tickets.

   b. For each band, write an algebraic equation that relates $p$, the class's profit, to $t$, the number of tickets sold.

   c. How many tickets must be sold for the Cautious Crew to be more expensive than the Muck Rockets? Justify your answer mathematically.

## Hiring a Popular Band

**Scoring Guide**

| Score | Description |
|---|---|
| **4** | The student demonstrates a thorough understanding of linear equations in two variables by correctly writing two linear equations in two variables and solving a real-world problem. |
| **3** | The student demonstrates a general understanding of linear equations in two variables by writing two linear equations in two variables and solving a real-world problem with only minor errors or omissions. The response indicates that the student could readily correct any errors and omissions if given written feedback. |
| **2** | The student demonstrates a basic understanding of linear equations by correctly completing a significant portion of the required tasks. The response indicates that the student would require some instruction to successfully complete the tasks. |
| **1** | The student demonstrates a minimal understanding of linear equations. The response indicates that the student would require significant instruction to complete the tasks. |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

**Sample Response:**

Part a: Cautious Crew Charge: $250 + .12(5 \times 300) = \$430$

$1500 - 430 = \$1{,}070$

Muck Rockets Charge: \$500

$1500 - 500 = \$1{,}000$

OR

CC: \$1070

MR: \$1000

Part b: Cautious Crew $p = 5t - (250 + 0.12[5t])$

OR

$p = 5t - (250 + 0.6t)$

OR

$p = 4.4t - 250$

Muck Rockets $p = 5t - 500$

## Hiring a Popular Band

Part c: $5t - 500 > 4.4t - 250$

$0.60t > 250$

$t > 416.\overline{6}$

417 tickets

OR

[Trial and Error solution that yields an answer of 417 tickets]

OR

$250 \div 0.6 \approx 417$

If selling 416 tickets, CC: \$1000.40, MR: \$1580

So, 417 tickets

417 tickets since CC makes less profit

OR

Muck Rockets: $5(417) - 500 = \$1585$

Cautious Crew: $5(417) - (250 + 0.6 \times 417) = \$1584.80$